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06MAT41

Fourth Semester B.E. Degree Examination, December 2010
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. Any missing data may be suitably assumed.

PART – A

1 a. Given $\frac{dy}{dx} + y - x^2 = 0$, $y(0) = 1$, $y(0.1) = 0.9052$, $y(0.2) = 0.8213$. Find correct to four decimal places $y(0.3)$ and $y(0.4)$ using modified Euler's method. **(07 Marks)**

b. Apply Runge – Kutta method of order four, to compute $y(2.0)$. Given $10\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, taking $h = 0.1$. **(07 Marks)**

c. The following table gives the solution of $\frac{dy}{dx} = x - y^2$. Find the value of y at $x = 0.8$, using Milne's predictor and corrector formulae.

X	0	0.2	0.4	0.6
Y	0	0.02	0.07	0.17

(06 Marks)

2 a. Show that polar forms of Cauchy's Riemann equation are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial V}{\partial \theta}, \frac{\partial V}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. **(07 Marks)**

b. Determine the analytic function $w = u + iv$ if $V = \log(x^2 + y^2) + x - 2y$. **(07 Marks)**

c. Find the Bilinear transformation which maps the points $z = 1, i, -1$ into $w = 0, 1, \infty$. **(06 Marks)**

3 a. State and prove Cauchy's integral formula. **(07 Marks)**

b. Find the Laurent series of $\frac{3x^2 - 6z + 2}{z^3 - 3z^2 + 2z}$. i) $1 < |z| < 2$ ii) $|z| > 2$. **(07 Marks)**

c. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where c is $|z| = 3$ using Cauchy's residue theorem. **(06 Marks)**

4 a. Solve the equation in series $\frac{d^2y}{dx^2} + x^2 y = 0$. **(07 Marks)**

b. Obtain the series solution of Bessel's differential equation in the form $y = AJ_n(x) + BJ_{-n}(x)$. **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$, find the value of a, b, c, d. **(06 Marks)**

PART – B

5 a. Fit a curve of form $y = ab^x$ and hence estimate y when x = 8.

X	1	2	3	4	5	6	7
Y	87	97	113	129	202	195	193

(07 Marks)

b. If θ is the angle between the lines of regression then show that

$$\tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right)$$ **(07 Marks)**

c. State and prove Baye's theorem. **(06 Marks)**

6 a. The pdf of a variate X is given by the following table :

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

For what value of k, this represents a valid probability distribution?
Also find : i) $P(x \geq 5)$ ii) $P(3 < x \leq 6)$. **(07 Marks)**

b. Given that 2% of the fuses manufactured by a firm are defective, find by using Poisson distribution, the probability that a box containing 200 fuses has
i) No defective fuses ii) 3 or more defective fuses iii) At least one defective fuse. **(07 Marks)**

c. The marks of 100 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) less than 65 ii) more than 75 iii) between 65 and 75. **(06 Marks)**

7 a. Explain the following terms :
i) Null hypothesis ii) Type I and type II error iii) Confidence limits. **(07 Marks)**

b. A sample of 100 days is taken from a coastal town of a certain district and of 10 of them are found to be very hot. What are the probable limits of the percentage of hot days in the district? **(07 Marks)**

c. A certain stimulus administrated to each of the 12 patients resulted in the following change in blood pressure.
5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 df = 2.201). **(06 Marks)**

8 a. The joint probability distribution of two random variables x and y is as follows :

x \ y	-2	-1	4	6
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine :
i) The marginal distribution of x and y ii) Co variance of x and y iii) Correlation of x and y. **(07 Marks)**

b. Verify that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}.$$

is a regular stochastic matrix. **(07 Marks)**

c. Explain:
i) Absorbing state of Markov chain ii) Transient state iii) Recurrent state. **(06 Marks)**

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06ME42B

Fourth Semester B.E. Degree Examination, December 2010
Mechanical Measurements and Metrology

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. What are the objectives of metrology? What is the necessity of standard for measuring system? **(06 Marks)**

b. What is an end standard. Using M112 standard slip gauge set, build 48.3275 mm. **(06 Marks)**

c. Briefly explain : i) International prototype meter ; ii) Reference standard ; iii) Wringing phenomenon ; iv) Significance of wavelength standard. **(08 Marks)**

2 a. Differentiate between : i) Hole basis and shaft basis system ; ii) Clearance fit and interference fit ; iii) Measuring device and gauge. **(06 Marks)**

b. What are the concepts of interchangeability and selective assembly? Which is advantageous? **(06 Marks)**

c. Determine the dimensions of shaft and hole for a fit $30\ H_8/f_7$. The given data are :
$i = 0.45\sqrt[3]{D} + 0.001D$
IT8 = 25i, IT7 = 16i.
Fundamental deviation for 'f' shaft $-5.5D^{0.41}$. Also design PLUG gauge to check the above hole. Take wear allowance as 10% gauge allowance. **(08 Marks)**

3 a. What are the needs and characteristics of comparator? **(06 Marks)**

b. What are the advantages of optical comparator over mechanical and pneumatic comparator? **(06 Marks)**

c. What is the principle by which an electrical comparator works? Explain briefly the construction and working of LVDT as a comparator. **(08 Marks)**

4 a. What is the principle of interferometry? How is it adopted in optical interferometer? **(06 Marks)**

b. What are the uses of : i) Sine center ; ii) Clinometer ; iii) Angle gauges. **(06 Marks)**

c. Explain briefly the feature that can be measured by optical flat and gear tooth vernier caliper. **(08 Marks)**

PART – B

5 a. Define : i) Calibration ; ii) Hysterisis ; iii) Loading effect. **(06 Marks)**

b. Differentiate : i) Sensor and transducer ; ii) Primary and secondary transducer ; iii) Accuracy and sensitivity. **(06 Marks)**

c. Explain the principle of resistance type and capacitive type electrical transducer. Name different types of electrical transducers. **(08 Marks)**

6 a. What is the necessity of modifying devices? What are the advantages of electrical modifying devices? **(06 Marks)**

b. Why an input circuitry is required? Explain briefly the ballast circuit. **(06 Marks)**

c. Explain briefly the working of analog electric meter indicator and x – y plotter. **(08 Marks)**

7 a. Briefly explain how pressure can be measured with elastic transducer. **(06 Marks)**

b. Explain one indirect method of force measurement. **(06 Marks)**

c. Explain briefly the working principle of eddy – current dynamometer. What are its advantages? **(08 Marks)**

8 a. Explain the working principles of radiation pyrometer and thermocouple. **(08 Marks)**

b. Write notes on : i) Thermocouple Material ; ii) Strain gauge factor ; iii) Strain gauge material. **(12 Marks)**

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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06ME43

Fourth Semester B.E. Degree Examination, December 2010
Applied Thermodynamics

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, selecting at least two from each part.***
2. Use of thermodynamic data hand book is permitted.

PART – A

1 a. Define the following: i) Adiabatic flame temperature ii) Stoichiometric air/ fuel ratio iii) Enthalpy of formation iv) Combustion efficiency v) Higher and lower calorific values **(10 Marks)**

b. A certain natural gas has the following volumetric analysis: 65 percent CH_4, 8 percent H_2, 18 percent N_2, 3 percent O_2, and 6 percent CO_2. This gas is now burned completely with the stoichiometric amount of dry air. Compute the air-fuel ratio and the analysis of products by volume. **(10 Marks)**

2 a. Show that the efficiency of air standard Brayton cycle is a function of isentropic pressure ratio. **(06 Marks)**

b. Sketch the ideal regenerative Brayton cycle in two stage compression and expansion with intercooling and reheat. Mark the points on the corresponding T-S diagram. (No Description) **(04 Marks)**

c. Consider an ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each stage of the compressor and turbine is 3. The air enters each stage of the compressor at 300 K and each stage of the turbine at 1200 K. Determine the back work ratio and the thermal efficiency of the cycle, when no regenerator is used. Assume an efficiency of 80 percent for each compressor stage and an efficiency of 85 percent for each turbine stage. **(10 Marks)**

3 a. Derive the expression for the air standard efficiency of an Otto cycle. **(06 Marks)**

b. Compare the efficiency of Otto and Diesel cycles for the same compression ratio and for the same state of air before compression. **(04 Marks)**

c. An ideal diesel engine has a compression ratio of 20 and uses air as the working fluid. The state of air at the beginning of the compression process is 95 kPa and 20°C. If the maximum temperature in the cycle is not to exceed 2200 K, determine (i) the thermal efficiency and (ii) the mean effective pressure. Assume constant specific heats for air at room temperature. **(10 Marks)**

4 a. Sketch the flow diagram and the corresponding temperature-entropy diagram of a reheat cycle and derive the expression for reheat cycle efficiency. What is the effect of reheat on i) specific output ii) cycle efficiency iii) steam rate and iv) heat rate of steam power plant? **(10 Marks)**

b. In a thermal power station, with a single reheat cycle, the steam at boiler outlet is at 8 MPa and 500°C. The reheating takes place at 3 MPa and the temperature at the end of reheat is the same as the boiler outlet temperature. If the condensate pressure is 20 kPa, calculate for the ideal process, using the Mollier chart, i) quality of steam at the turbine exhaust ii) work done by the pump ii) work done by the turbine iv) cycle efficiency v) steam rate. **(10 Marks)**

PART – B

5 a. Derive the condition for minimum work output to a two stage reciprocating air compressor with perfect intercooling. What are the assumptions you made? **(10 Marks)**

5 b. In a single acting two stage reciprocating air compressor, 4.5 kg of air per minute is compressed from 1.013 bar and 15°C through a pressure ratio of 9 to 1. Both stages have the same pressure ratio and the path of compression and expansion in both the stages is $PV^{1.3}$ = constant. If the intercooling is complete, calculate the indicated power and cylinder swept volume required. Assume that the clearance volume of both the stages is 5 % and the compressor runs at 280 rpm. **(10 Marks)**

6 a. Explain the working of a vapour absorption refrigeration system. **(10 Marks)**

b. A refrigerator uses R-134 a as the working fluid and operates on an ideal vapor-compression refrigeration cycle between 0.12 and 0.7 MPa. The mass flow rate of the refrigerant is 0.05 kg/s. Show the cycle on a T-s diagram with respect to saturation lines. Determine (i) the rate of heat removal from the refrigerated space and the power input to the compressor, (ii) the rate of heat rejection to the environment, and (iii) the coefficient of performance.

Properties of R-134a

Absolute pressure (kPa)	Saturation temperature°C	Enthalpy kJ/kg		Entropy kJ/kg	
		Saturated Liquid	Saturated Vapour	Saturated Liquid	Saturated Vapour
120	-22.32	22.29	236.97	0.09275	0.94779
700	26.69	88.82	265.03	0.33230	0.91994

Take vapour specific heat at 700 kPa = 1.0243 kJ/kgK. **(10 Marks)**

7 a. Define the following: i) Dry bulb temperature ii) Dew point temperature iii) Specific Humidity iv) Adibatic saturation temperature v) Psychrometrics. **(10 Marks)**

b. Air enters at 32°C and relative humidity of 70 % in a summer air conditioning system where the air is cooled and then dehumidified. The air leaving the cooling coil is saturated at the coil temperature. It is then heated to comfort condition of 24 C and 50 % relative humidity. Sketch the flow diagram of the system and represent the various processes in the skelton of psychrometric chart. Determine i) the temperature of the cooling coil ii) the amount of moisture removed per kg of dry air in the cooling coil iii) the heat removed per kg dry air in the cooling coil and iv) the heat added per kg dry air in the heating coil. **(10 Marks)**

8 a. Describe the Morse test. How can it be used for finding the friction power and the indicated power of an IC Engine? **(08 Marks)**

b. Morse test is conducted on a four stroke four cylinder petrol engine at a constant speed and the following power is measured :

With all cylinders working = 15.6 kW
With number 1 cylinder cut off =11.1 kW
With number 2 cylinder cut off = 11.3 kW
With number 3 cylinder cut off = 10.8 kW
With number 4 cylinder cut off = 11.0 kW

The bore and stroke of each cylinder is 75 mm and 100 mm respectively. The clearance volume of the cylinder is 100cc. The fuel is consumed at the rate 6 kg/hr. If the calorific value of the fuel is 42000 kJ/kg. Determine i) Indicated power ii) Frictional Power iii) Mechanical Efficiency iv) Brake thermal efficiency v) Relative efficiency with respect to brake thermal efficiency. **(12 Marks)**

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06ME45

Fourth Semester B.E. Degree Examination, December 2010
Manufacturing Processes – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Sketch and explain the nomenclature of a single point cutting tool, highlighting the significance of different angles. **(10 Marks)**

b. Explain the following types of tool wear with necessary sketches : i) crater wear ; ii) Flank wear. **(10 Marks)**

2 a. Explain the different properties required of a cutting tool material. **(08 Marks)**

b. Explain the following cutting tool materials : With respect to usage, composition and structure. i) High speed steel ; ii) Ceramics. **(12 Marks)**

3 a. With a sketch, explain any two methods of taper turning, using a Lathe. **(12 Marks)**

b. With a sketch, explain one method of quick return mechanism of a shaper. **(08 Marks)**

4 a. Sketch and explain a radial drilling machine, highlighting its field of applications. **(10 Marks)**

b. Sketch and explain the following drilling operation with their field of application.: i) Counter sinking ; ii) Counter boring, **(10 Marks)**

PART – B

5 a. Differentiate between a plain and an universal milling machine. **(05 Marks)**

b. Explain with sketches, any three of the following :
   i) Straddle milling
   ii) Gang milling
   iii) Face milling
   iv) Up milling and down milling. **(15 Marks)**

6 a. Sketch and explain a centreless grinding wheel, highlighting its advantages and disadvantages. **(10 Marks)**

b. Explain the different factors to be considered, in the selection of a grinding wheel. **(10 Marks)**

7 Sketch and explain the following machines, indicating their field of applications :
   i) Lapping machine **(10 Marks)**
   ii) Honing machine. **(10 Marks)**

8 Write explanatory notes on any two of the following processes :
   i) Electro chemical machining
   ii) Laser beam machining
   iii) Abrasive jet machining. **(20 Marks)**

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06ME44

Fourth Semester B.E. Degree Examination, December 2010
Kinematics of Machines

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. Missing data, if any, may be assumed suitably.
3. Graphical solutions can be drawn on the answer book/graphs sheet in the answer book.

PART – A

1 a. Distinguish between :
i) Higher pair and lower pair
ii) Kinematic pair and kinematic chain. **(08 Marks)**

b. Sketch and explain the single slider crank mechanism. Explain with a neat sketch, any two of its inversions. **(12 Marks)**

2 a. Differentiate between the following, with examples :
i) Constrained motion and unconstrained motion
ii) Machine and mechanism **(08 Marks)**

b. Sketch and explain the working of an elliptical trammel. Prove that it traces an ellipse. **(12 Marks)**

3 a. State and prove Kennedy's theorem of instantaneous centre. **(08 Marks)**

b. In the mechanism shown in Fig.Q.3(b) the crank AB rotates at 200 rpm. Find the velocities of C, D, E, F and P. Also find the acceleration of the slider at C. The dimensions of the various links are; AB = 12 cm, BC = 48 cm, CD = 18 cm, DE = 36 cm, EF = 12 cm and FP = 36 cm. **(12 Marks)**

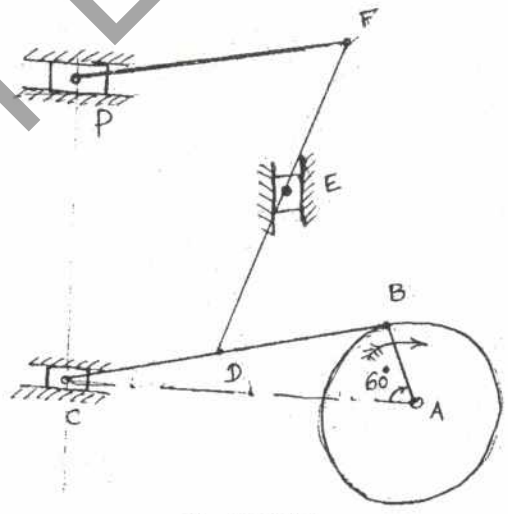

Fig.Q.3(b).

4 a. Write a note on Coriolis component of acceleration. **(08 Marks)**

b. The lengths of the crank and the connecting rod of a reciprocating engine are 200 mm and 800 mm respectively. The crank is rotating at an uniform speed of 480 rpm. Using Klein's construction, find : i) Acceleration of the piston ; ii) The acceleration of the middle point of the connecting rod and iii) Angular acceleration of the connecting rod, when the crank has turned through 45° from the inner dead centre. **(12 Marks)**

PART – B

5 a. Obtain the loop closure equation, for a four bar mechanism. **(08 Marks)**

b. Develop an equation for the relationship between the angular velocities of the input crank and output crank of the four bar linkage shown in Fig.Q.5(b), using loop closure equation. **(12 Marks)**

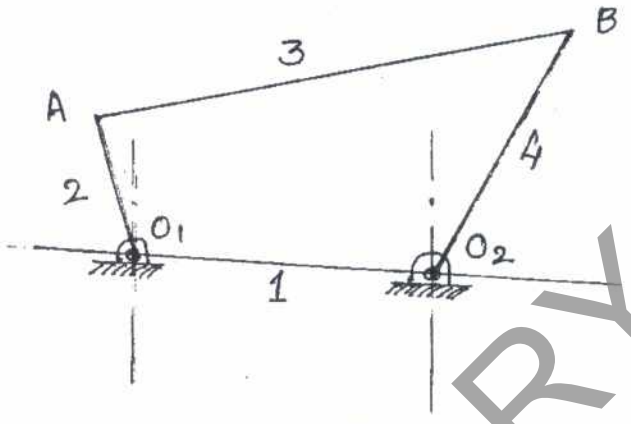

Fig.Q.5(b).

6 a. State and prove the "Law of gearing". **(06 Marks)**

b. Derive an expression for the 'arc of contact'. **(06 Marks)**

c. A pair of gears has 16 teeth and 18 teeth, a module 12.5 mm, addendum 12.5 mm and a pressure angle 14.5 degrees. Prove that the gears have interference. Determine the minimum number of teeth and the velocity ratio to avoid the interference. **(08 Marks)**

7 a. Explain with a sketch the "Differential mechanism" of an automobile. **(05 Marks)**

b. In an epicycle gear train, the internal wheels A and B and the compound wheels C and D, rotate independently about the axis 'O'. The wheels E and F rotate on the pins fixed to the arm G. The wheel E gears with A and C and the wheel F gears with B and D. All the wheels have the same module. The number of teeth are $T_C = 28$ $T_D = 26$ $T_E = T_F = 18$

i) Sketch the arrangement

ii) Find the number of teeth on the wheels A and B

iii) If the arm G rotates at 100 rpm clockwise and A is fixed, find the speed of B

iv) If the arm G rotates at 100 rpm clockwise and wheel A makes 10 rpm, counterclockwise find the speed of B. **(15 Marks)**

8 Draw the cam profile for the following details :

Minimum radius of cam = 30 mm

Roller follower with roller = 10 mm radius,

Axis of the roller is along the straight line with the axis of the cam shaft.

$\theta_{rise} = 90^o$ lift = 30 mm, UARM

$\theta_{dwell} = 30^o$ in lifted position

$\theta_{return} = 120^o$ SHM

θ_{dwell} for remaining portion of the cam rotation. **(20 Marks)**

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06ME46B

Fourth Semester B.E. Degree Examination, December 2010
Fluid Mechanics

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. Assume suitable data, if required.

PART – A

1 a. Differentiate between gauge pressure and absolute pressure. Represent positive and negative gauge pressures on a chart. **(03 Marks)**

b. Give reasons for the following :
i) Viscosity changes with temperature rise.
ii) Mercury (H_g) is preferred as a manometric liquid.
iii) Free surface of water in a capillary tube is concave.
iv) Light weight objects can float on the free surface of liquids.
v) Metacentric height is positive for stable equilibrium of floating bodies. **(10 Marks)**

c. Derive the relation for capillary rise of water in a glass tube. **(03 Marks)**

d. A liquid bubble of 2cm radius has an internal pressure of 12.95 Pascals. Determine the surface tension of the liquid film. **(04 Marks)**

2 a. Derive the relations for hydrostatic forces on a curved surface, which is immersed in a liquid of specific weight 'W'. **(06 Marks)**

b. With a neat sketch, explain the working of an inverted u – tube manometer. **(06 Marks)**

c. A wooden block of size 6m x 4m x 2m floats on fresh water. Depth of immersion of the wooden block is 1.2 m. A concrete block is placed centrally on the surface of the wooden block, so that,
i) The top surface of the wooden block touches the free surface of water
ii) Both wooden block and concrete block submerge completely in water.
Assume specific gravity of concrete = 2.5. Find the volume of the concrete block in each case. **(08 Marks)**

3 a. Derive the continuity equation for a three dimensional flow, in Cartesian co-ordinates. **(08 Marks)**

b. Show that the streamlines and equipotential lines are orthogonal to each other. **(04 Marks)**

c. A stream function represents 2-D fluid flow, $\psi = 2xy$. Find the velocity at a point P(3, 4). Check whether the flow is rotational. Find the velocity potential function ϕ. **(08 Marks)**

4 a. Mention the applications of model similitude. **(02 Marks)**

b. Explain the significance of non – dimension numbers.
i) Mach number ; ii) Froude's number ; iii) Weber number ; iv) Reynolds' number. **(08 Marks)**

c. Using Buckingham π - theorem, show that the velocity of fluid flow through a circular orifice is given by $V = \sqrt{2gH}\ \phi\left(\frac{D}{H}, \frac{\mu}{\rho VH}\right)$, where

H = Head of fluid flow ; D = Diameter of the orifice
μ = Dynamic viscosity of the fluid ; ρ = Density of the fluid.
g = gravitational acceleration. **(10 Marks)**

PART – B

5 a. Derive the Bernoulli's equation for a steady, incompressible fluid flow. List the assumptions. Mention the significance of each term in Bernoulli's equation. **(10 Marks)**

b. Pipeline AB carries oil of specific gravity 0.90. Diameter of the pipe at A is 250 mm and that at B is 500 mm. End B of the pipe is 6 meters above the end A. The pressure intensities at A and B are 200 kN/m^2 and 120 kN/m^2 respectively. Discharge of oil is 450 lit/sec. Determine : i) Loss of head and ii) Direction of oil flow. **(10 Marks)**

6 a. Differentiate between a venturimeter and an orificemeter. **(04 Marks)**

b. A pitot – tube is used for measuring the velocity of air flow through a duct. A u – tube water manometer shows a deflection of 12 mm of water. If the coefficient of pitot tube is 0.98, find velocity of air flow and mass flow rate of air. Assume specific weight of air as 10 N/m^3. Diameter of the duct is 500 mm. **(06 Marks)**

c. Oil of specific gravity 0.90 flows through an inclined venturimeter. Inlet and throat diameters are 30 cm and 15cm respectively and the throat is 30cm above the inlet section. Pressure intensity at the inlet is 150 kPa and deflection in mercury manometer is 25 cm. Determine the rate of oil flow in lts/sec and also the pressure intensity at the throat. Assume $C_d = 0.98$ for the venturimeter. **(10 Marks)**

7 a. Derive a relation for the discharge through a circular pipe of diameter D, for the viscous flow. **(08 Marks)**

b. A 100 meters long pipeline connects two reservoirs. The difference in waterlevels is 15 meters. The pipeline has two equal sections of 50 meters each. Diameters of first and second sections are 25 mm and 50 mm respectively. If the friction coefficient of pipe material is 0.005, determine the velocity of water flow through the two sections and the rate of water flow in litres/sec. Represent TEL and HGL. **(12 Marks)**

8 a. Define drag force and lift force. **(04 Marks)**

b. Define and explain :

i) Boundary layer thickness

ii) Mach cone, Mach angle

iii) Subsonic flow. **(08 Marks)**

c. A projectile travels in air of pressure 1.01 × 10^5 N/m^2 at 10°C. Speed of projectile is 1500 km/hour. Determine the Mach number and the Mach angle. Assume k = 1.4 and R = 287 J/kg k. **(08 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, December 2010

Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the ratio in which the line joining (2, 4, 16) and (3, 5, –4) is divided by the plane $2x-3y+z+6=0$. **(06 Marks)**

b. Find the angle between the lines whose direction cosines are given by $3l+3+5n=0$ and $6mn-2ln+5lm=0$. **(07 Marks)**

c. Derive the equation of the plane in the form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. **(07 Marks)**

2 a. Find the reflection of the point (1, 3, 4) in the plane $2x-y+z+3=0$. **(07 Marks)**

b. Find the equation of the line through (1, 2, -1) and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$. **(06 Marks)**

c. Prove that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{1+z}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the coordinates of their point of intersection. **(07 Marks)**

3 a. If $\vec{A}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{B}=3\hat{i}-\hat{j}+2\hat{k}$ then :

i) Show that $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are orthogonal and

ii) Find the angle between $2\vec{A}+\vec{B}$ and $\vec{A}+2\vec{B}$. **(07 Marks)**

b. Prove that $[\vec{A}+\vec{B},\vec{B}+\vec{C},\vec{C}+\vec{A}]=2[\vec{A}\,\vec{B}\,\vec{C}]$. **(06 Marks)**

c. If $\vec{A}=2\hat{i}-\hat{j}+3\hat{k}$, $\vec{B}=-\hat{i}+3\hat{j}+3\hat{k}$ and $\vec{C}=\hat{i}+\hat{j}-2\hat{k}$, find the reciprocal triad $(\vec{A}'\vec{B}'\vec{C}')$. **(07 Marks)**

4 a. For the curve $\vec{R}=a(\cos t\,\hat{i}+\sin t\,\hat{j}+t\tan\alpha\,\hat{k})$ where a and α are constants, evaluate $\left|\frac{d\vec{R}}{dt}\times\frac{d^2\vec{R}}{dt^2}\right|$. **(06 Marks)**

b. The position vector of a moving particle at time t is $\vec{R}=t^2\hat{i}-t^3\hat{j}+t^4\hat{k})$. Find the tangential and normal components of its acceleration at t = 1. **(07 Marks)**

c. Find the directional derivative ϕ = xyz along the direction of the normal to the surface $x^2z+y^2x+z^2y=3$ at the point (1, 1, 1). **(07 Marks)**

5 a. Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. **(07 Marks)**

b. If $\vec{F} = e^{xyz}(\hat{i} + \hat{j} + \hat{k})$ find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$. **(06 Marks)**

c. Prove that $\nabla \times \nabla \times \vec{F} = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}$. **(07 Marks)**

6 a. Prove that $L\{\cos at\} = \dfrac{s}{s^2 + a^2}$ $s > 0$ **(05 Marks)**

b. Find : i) $L\{e^{-t}\sin^2 t\}$ ii) $L\{te^{-t}\sin 3t\}$ iii) $L\left\{\dfrac{\cos 2t - \cos 3t}{t}\right\}$ **(15 Marks)**

7 a. IF $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ t - 1, & 2 < t < 3 \\ 7, & t > 3 \end{cases}$, find $L\{f(t)\}$. **(07 Marks)**

b. Find $L^{-1}\left\{\dfrac{4s+5}{(s-1)^2(s+2)}\right\}$. **(06 Marks)**

c. Apply convolution theorem to evaluate $L^{-1}\left\{\dfrac{s}{(s^2 + a^2)^2}\right\}$. **(07 Marks)**

8 a. If $f'(t)$ is a continuous function and $L\{f(t)\} = F(s)$ then prove that $L\{f'(t)\} = sF(s) - f(0)$. **(04 Marks)**

b. Solve the following using Laplace transform :
$y'' + 2y' - 3y = \sin t$, when $y(0) = 0 = y'(0)$. **(06 Marks)**

c. Using Laplace transform method, solve the simultaneous equations:
$\dfrac{dx}{dt} + 5x - 2y = t$; $\dfrac{dy}{dt} + 2x + y = 0$, given $x = y = 0$, when $t = 0$. **(10 Marks)**

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